FDTD simulations of TF/SF plane waves in the presence of PEC scatterers

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Abstract

FDTD (Finite Difference Time Domain) computer simulations of the electromagnetic field are often used to simulate field propagation of antenna radiation, to calculate radar cross section, and in design of photonic and microwave structures. TF/SF (Total Field/Scattered Field) plane waves are efficient choice for field/wave propagation in one defined direction. Basic concept of TF/SF technique and FDTD implementation of TF/SF plane waves are presented in this paper. The numerical results obtained by 2D and 3D FDTD computer simulations with TF/SF waves and the Ricker source function for PEC cylinder and PEC sphere in free space are given and discussed.

1 Introduction

In modern day CE (computational electromagnetics) there is a constant need for faster calculation processes and method improvements and to maximize hardware utilizations. There is a wide range of numerical methods in CE, however one of the most commonly used numerical technique for simulation of complex and largely inhomogeneous structures is Finite Difference Time Domain (FDTD) method, due to its straightforward approach. The simulation of plane waves are very useful in CE. There are many practical electromagnetic problems which can be solved by simulating plane wave propagation, such as calculation of radar cross section or calculation of field from most antennas (which can be, after a distance on the order of tens of wavelengths, approximated as a plane wave). There are many ways to generate plane waves in FDTD method and one of the most effective are TF/SF plane waves.

In this paper the application of TF/SF plane waves in FDTD simulations is examined. Basic theory of TF/SF technique and its implementation in FDTD method is given and some advantages of TF/SF plane waves are pointed out. Numerical results of performed 2D and 3D FDTD simulation of the electromagnetic field propagation in free space with PEC cylinder and PEC sphere are presented and discussed.

2 FDTD method

FDTD method is powerful numerical method used for numerical simulation of the electromagnetic field propagation in CE. Practical usage of this method is in antennas design, high-speed digital and microwave circuits design, and design of wireless communications and integrated optical devices. The considerable advancements within the FDTD technique have been achieved and reported in the literature over the past few years, mainly due to the increased computing power of today's computers.

FDTD method is based on the direct spatial and time discretization of the Maxwell equations in time domain [1, 2]. There is various number of algorithms used in FDTD method. Almost all of them are based on rectangular Cartesian grid originally proposed by Yee in 1966 [1].

In this paper we have used standard explicit FDTD algorithm. Electromagnetic field is propagated in free space with *Perfect electric conductor* (PEC) placed in the middle of the computational domain. PEC is material with infinite electrical conductivity used as a scatterer for the incident field. FDTD computational domain is finished with Mur's second-order boundary condition [8].

The pulse sources are widely used in FDTD simulations due to their ability to generate wide range of frequencies. The field from these sources propagates in all directions, which in some cases can be considered as their disadvantage. There are certain electromagnetic problems when the propagation in only one direction is desirable. One of the most effective ways to accomplish that is to use TF/SF plane waves.

3 TF/SF plane waves

The total-field / scattered-field (TF/SF) technique is developed to achieve a plane-wave source propagating in one direction of the FDTD computational domain. This technique, described in [3] for the first time, is an effective way of introducing energy into FDTD simulation, i.e. into computational domain, generating uniform plane waves of arbitrary polarization and time dependence. The TF/SF formulation is based on linearity of the Maxwell equations [2]. We assume that total electric and magnetic fields E_{tot} and H_{tot} can be decomposed as:

$$E_{tot} = E_{inc} + E_{scat}, \qquad H_{tot} = H_{inc} + H_{scat}, \tag{1}$$

with the assumption that the incident fields E_{inc} and H_{inc} are known at all points of the computational domain, and scattered-wave fields E_{scat} and H_{scat} are initially unknown.

TF/SF concept is defined by dividing the computational domain (grid) with boundary, identified as the TF/SF boundary. The TF/SF boundary is fictitious boundary which divides FDTD domain into two regions: a total field (TF) region (contains incident and scattered fields) and a scattered field (SF) region (contains scattered fields only). To enable the efficiency of Yee's algorithm, having in mind that TF/SF technique is based on the linearity of the Maxwell equations, the update equations of the fields in both regions must be consistent. Consistency is fulfilled at the TF/SF boundary, by adjusting the field at the nodes adjacent to the boundary. For the total-field nodes of TF region, adjacent to the boundary, one of more neighboring nodes are missing because they are supposed to be placed in the SF region, at the other side of the boundary. Considering that in SF region the incident field components does not exist, TF/SF boundary adjusts the update equation by adding missing nodes. On the other hand, TF/SF boundary subtracts neighboring nodes which lie in TF region, while updating SF nodes. Detailed implementation of TF/SF boundary can be found in [2]. Yee's scheme with TF/SF boundary is presented in Figure 1.

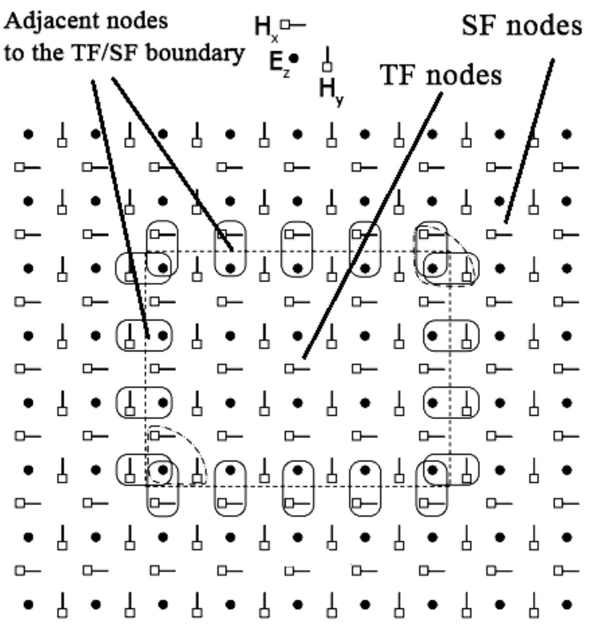

Figure 1. Illustration of Yee's scheme with TF/SF boundary

Although, in this paper, we only considered using TF/SF boundary to introduce plane waves, it is possible to generate any type of field. If, for example, incident field is to be generated by a dipole source which is located physically outside of the grid, such a field can be introduced over the TF/SF boundary. The description of the field over the boundary should match the way the field actually behaves in the grid. This can cause certain amount of leakage across the boundary, which can be reduced by finer discretization of the grid.

4 Results

In order to examine benefits of the TF/SF plane wave implementation in FDTD method, *C++* codes are written. The explicit FDTD algorithm is used. Numerical results of the electromagnetic field FDTD simulations are plotted with the command-line driven *gnuplot* graphing utility.

It is well known that the FDTD field simulations in photonics and microwaves sometimes require thousands and thousands time steps. In FDTD simulations performed in this paper we choose the time-length of simulations to be 600 time steps. There is no need for longer simulations if we wish to show only the benefits of TF/SF plane wave usage in FDTD method.

In subsection 4.1 the numerical results of the 2D FDTD simulation of the TF/SF plane wave propagation in free space and the field scattering from PEC cylinder are presented. In subsection 4.2 the numerical results of the 3D FDTD simulation of the TF/SF plane wave propagation in free space and the field scattering from PEC sphere are shown. The Ricker pulse is used as incident pulse of the TF/SF plane wave in both FDTD simulations.

4.1 2D FDTD simulation of the electromagnetic field in the presence of PEC cylinder

In this 2D FDTD simulation of the electromagnetic field, TM mode TF/SF plane wave is propagated in free space with the PEC scatterer in the center of the computational window. Plane wave is *z* polarized and propagated in *x* direction. Incident pulse is the Ricker pulse [4], with the source function [5-7]:

$$f_r(q) = \left(1 - 2\pi^2 \left[\frac{C_n q}{N_P} - \mathrm{M}\right]^2\right) e^{-\pi^2 \left[\frac{C_n q}{N_P} - \mathrm{M}\right]^2}, \quad (2)$$

where $C_n=c\Delta t/\Delta x$ represents *Courant number* (and it is set to $C_n=1/\sqrt{2}$), Δt is time step, Δx is spatial step, c is the speed of the wave propagation, q is time constant, $N_p=\lambda_P/\Delta x$ gives points per wavelength at the peak frequency, needed for the pulse width definition, λ_P - the plane wave wavelength. M is arbitrary time constant which is important for time delay. In all simulations in this paper was set: $M=1$ and $N_p = 30$.

The FDTD computational domain is chosen to be 400 by 400 spatial cells, while the simulation time is chosen to be 600 time steps. TF region is placed from the 30th to the 370th spatial step in *x* direction and from the 30th to the 370th special step in *y* direction. SF region is placed from the 371th to the 400th spatial step in both directions. FDTD computational domain is finished with Mur's second-order absorbing boundary condition [8]. TF/SF plane wave is generated at the left side of the TF/SF boundary. PEC scatterer is a cylinder with radius of 70 spatial cells and it is located at the center of the computational domain, hence at the center of TF region.

The field component E_z in the transverse plane, TM mode, after 120, 360 and 480 time steps of FDTD simulation, is shown in Figure 2, Figure 3 and Figure 4, respectively.

After 120 time steps the TF/SF plane wave is fully generated, and the field component $E_{z\,inc}$ is displayed in Figure 2. It is shown that the incident wave is limited only to TF region and that the field propagates only in the *x* direction. Figure 2 illustrates that there is no other, unwanted, field components.

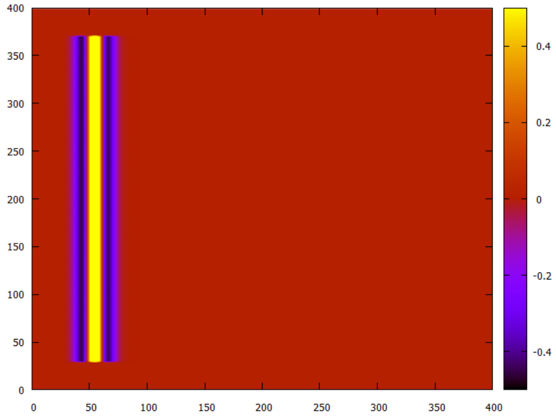

Figure 2 E_z field component in the transverse plane, TM mode, after 120 time steps

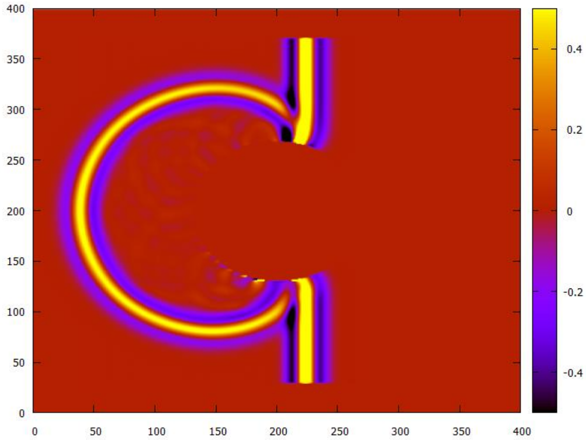

Figure 3 E_z field component in the transverse plane, TM mode, after 360 time steps

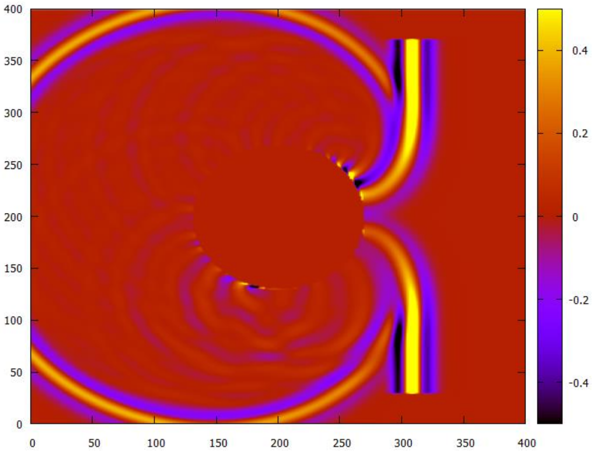

Figure 4 E_z field component in the transverse plane, TM mode, after 480 time steps

The wave reflection from PEC cylinder is shown in Figure 3 and Figure 4. It can be seen that scattered components of the reflected wave ($E_{z\ scat}$) propagate in the SF region, while incident wave components ($E_{z\ inc}$) remain in the TF region until the end of the FDTD simulation.

4.2 3D FDTD simulation of the electromagnetic field in the presence of PEC sphere

In the 3D FDTD simulation of the electromagnetic field TF/SF plane waves are propagated in free space, with PEC sphere in the center of the FDTD computational domain. The incident pulse is the Ricker pulse, with the same parameters as in previous simulations, except *Courant number*, which is set to $C_n=1/\sqrt{3}$. The incident field is polarized in *z* direction and propagate in *x* direction. Dimension of FDTD computational domain is: 200 by 200 by 200 spatial cells and the simulation time-length is 600 time steps. TF region is placed from the 20th to the 180th spatial step and SF region is placed from the 181th to the 200th spatial step in all three directions. Mur's second-order absorbing boundary conditions [8] are used at the boundaries. TF/SF plane wave is generated on the left-hand side of the TF/SF boundary, Figure 2. PEC scatterer is a sphere with radius of 35 spatial cells located in the center of the FDTD computational domain, hence in the center of the TF region.

The field component E_z in the transverse plane, calculated at *y* plane, is displayed in Figure 5, Figure 6 and Figure 7. Wave propagating in *x* direction is shown after 100, 190 and 320 time steps. After 100 time steps the TF/SF plane wave is generated, and that field is shown in Figure 5. It can be seen from the Figure 5 that the incident field exist in TF region only, and the propagation is directed in only one: *x* direction.

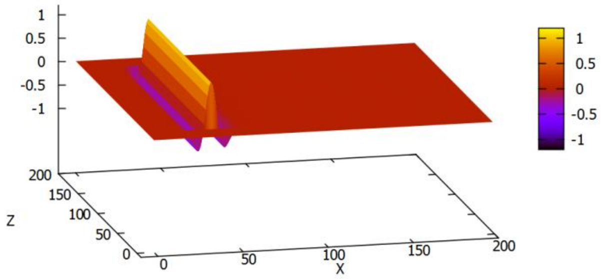

Figure 5 E_z field component in the transverse plane, at *y=const* plane, after 100 time steps

The moment of the first interaction of TF/SF plane wave and PEC sphere is shown in Figure 6. The reflected component of the field propagates in the opposite direction to the incident wave.

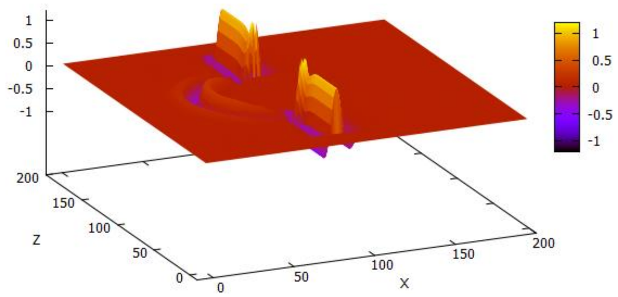

Figure 6 E_z field component in the transverse plane, at *y=const* plane, after 190 time steps

The moment when the plane wave passes PEC sphere is shown in Figure 7. It is important to notice that the incident field components ($E_{z\ inc}$) still remain only in the TF region, with scattered field components as well ($E_{z\ scat}$). In the SF region only scattered field components ($E_{z\ scat}$) propagate as it was expected.

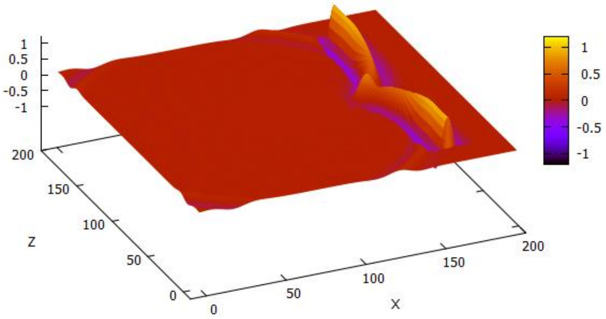

Figure 7 E_z field component in the transverse plane, at *y=const* plane, after 320 time steps

5 Conclusion

In this paper we explained and examined basic concepts of TF/SF plane waves and their implementation in FDTD method throughout 2D and 3D FDTD simulations. TF/SF plane waves with the Ricker source function are numerically propagated in free space with scatterers in the middle of the computational window: PEC cylinder for 2D case and PEC sphere for the 3D case. Numerical results, graphically shown, clearly indicate effectiveness of using TF/SF plane waves in solving 2D and 3D FDTD problems in CE.

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