Floating potential, sheath potential drop and sheath thickness of a planar electrode immersed in a magnetized plasma with oblique magnetic field

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Abstract

A one-dimensional fluid model is presented and used for the analysis of the potential formation in front of a negative planar electrode immersed in magnetized plasma. The Bohm criterion and the floating condition are derived and used to determine the sheath thickness and the sheath potential drop. With increasing angle of magnetic field lines with respect to the electrode surface the floating potential, the sheath edge potential and the sheath potential drop all increase, while the sheath thickness has a minimum.

1 Introduction

Problem of the sheath formation in front of a negative electrode immersed in magnetized plasma is an important one and has attracted a lot of attention. The pioneering works of Chodura [1], Riemann [2,3], Stangeby [4] and some others [5,6] have been extended and revisited in many directions. In this work attention is focused to the effect of ion temperature and magnetic field angle to the floating potential of a planar electrode immersed in plasma where magnetic field lines form an arbitrary angle with the electrode surface. The study is aimed at a fusion application. The scrape-off layer of a tokamak, for example has low density plasma in strong magnetic fields. Further, the angle of incidence of the magnetic field lines onto the divertor spans nearly the whole range from perpendicular to parallel. In the next section a one-dimensional fluid model is described. In section 3 results of the model are presented and finally some conclusions are given.

2 Model

The ions are assumed to obey the continuity equation and equation of motion:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot \left(n_i \mathbf{u}_i \right) = S_i , \qquad (1)$$

$$m_i n_i \left(\frac{\partial \mathbf{u}_i}{\partial t} + \left(\mathbf{u}_i \cdot \nabla \right) \mathbf{u}_i \right) = = n_i e_0 \left(\mathbf{E} + \mathbf{u}_i \times \mathbf{B} \right) - \nabla p_i + \mathbf{A}_i - m_i \mathbf{u}_i S_i . \qquad (2)$$

Here m_i is the ion mass, t is time, $\mathbf{u}_i$ is the ion fluid velocity, n_i is the ion density, e_0 is the elementary charge, $\mathbf{E}$ is electric field, $\mathbf{B}$ is magnetic field, p_i is the ion pressure, S_i is the source term and $\mathbf{A}_i$ is the collision term. Detailed descriptions of elastic collisions in fluid models can be found in e. g. [7]. In this work we simply take it to be proportional to ion fluid velocity $\mathbf{u}_i$:

$$\mathbf{A}_i = -m_i n_0 \nu \mathbf{u}_i . \qquad (3)$$

As for the source term S_i it is assumed that the predominant ionization mechanism is ionizing collisions of electrons and neutral atoms. The electrons are assumed to be Boltzmann distributed:

$$n_e \left(\mathbf{r} \right) = n_0 \exp \left(\frac{e_0 \Phi \left(\mathbf{r} \right)}{k T_e} \right). \qquad (4)$$

The source term is therefore given by:

$$S_i = \frac{n_0}{\tau} \exp \left(\frac{e_0 \Phi}{k T_e} \right). \qquad (5)$$

Here k is the Boltzmann constant, T_e is the electron temperature, ν is the frequency of elastic collisions, τ is average time between two consecutive ionizing collisions between electron and a neutral atom and n_0 is the plasma density in the plasma region which is not perturbed by the electrode – this means beyond the pre-sheath. The potential Φ is determined by the Poisson equation:

$$\nabla^2 \Phi \left(\mathbf{r} \right) = -\frac{e_0}{\varepsilon_0} \left(n_i \left(\mathbf{r} \right) - n_e \left(\mathbf{r} \right) \right). \qquad (6)$$

Here ε_0 is the permittivity of the free space. Furthermore it is assumed that the following equation of state relates the ion pressure, p_i, density n_i and temperature T_i:

$$p_i = \kappa n_i k T_i . \qquad (7)$$

Here κ is the polytropic coefficient. Our model is isothermal so the ion temperature is assumed to be constant everywhere and therefore $\kappa = 1$. Our model is one-dimensional. The x axis is assumed to be perpendicular to the electrode. The gradient and Laplace operators are replaced by derivatives over x

$$\nabla \to \frac{\mathrm{d}}{\mathrm{d}x}, \ \nabla^2 \to \frac{\mathrm{d}^2}{\mathrm{d}x^2}, \ \nabla p_i \to \frac{\mathrm{d}p_i}{\mathrm{d}x},$$

The electric field has only one component E_x which is given by:

$$E_x = -\frac{\mathrm{d}\Phi}{\mathrm{d}x}.$$

The magnetic field lies in the xy plane forming the angle α with the y axis, as shown in Fig. 1.

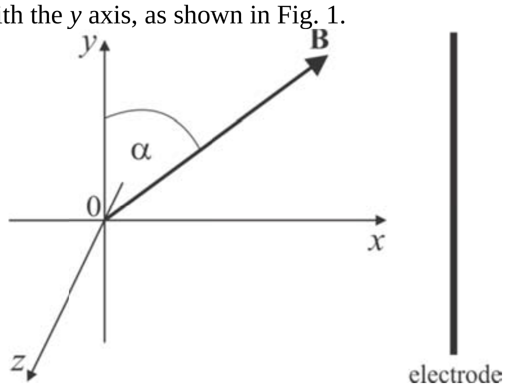

Figure 1: Schematic of the model. The magnetic field lies in the xy plane and the x axis is perpendicular to the electrode.

The components of the magnetic field are

$$\mathbf{B} = B\left(\sin\alpha,\ \cos\alpha,\ 0\right).$$

The equations (1), (2) and (6) are therefore written in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}X}\left(nV_x\right) = \varepsilon\exp\left(\Psi\left(X\right)\right), \tag{8}$$

$$\begin{gathered} nV_x\frac{\mathrm{d}V_x}{\mathrm{d}X} + n\frac{\mathrm{d}\Psi}{\mathrm{d}X} + \Theta\frac{\mathrm{d}n}{\mathrm{d}X} + \\ + K\varepsilon\cos\alpha nV_z + \varepsilon V_x\left(\exp\left(\Psi\right) + Z\right) = 0, \end{gathered} \tag{9}$$

$$nV_x\frac{\mathrm{d}V_y}{\mathrm{d}X} - K\varepsilon\sin\alpha nV_z + \varepsilon V_y\left(\exp\left(\Psi\right) + Z\right) = 0, \tag{10}$$

$$\begin{gathered} nV_x\frac{\mathrm{d}V_z}{\mathrm{d}X} + K\varepsilon\left(\sin\alpha nV_y - \cos\alpha nV_x\right) + \\ + \varepsilon V_z\left(\exp\left(\Psi\right) + Z\right) = 0, \end{gathered} \tag{11}$$

$$\frac{d^2\Psi}{dX^2} = \exp\Psi - n. \tag{12}$$

The following variables have been introduced:

$$\begin{gathered} \lambda_D = \sqrt{\frac{\varepsilon_0 kT_e}{n_0 e_0^2}},\ c_0 = \sqrt{\frac{kT_e}{m_i}},\ K = \frac{e_0 B\tau}{m_i},\ \mu = \frac{m_e}{m_i}, \\ n = \frac{n_i}{n_0},\ \Psi = \frac{e_0\Phi}{kT_e},\ \Theta = \frac{T_i}{T_e},\ \varepsilon = \frac{1}{\tau}\sqrt{\frac{\varepsilon_0 m_i}{n_0 e_0^2}}, \\ Z = \nu\tau,\ V_x = \frac{u_{ix}}{c_0},\ V_y = \frac{u_{iy}}{c_0},\ V_z = \frac{u_{iz}}{c_0},\ X = \frac{x}{\lambda_D}. \end{gathered} \tag{13}$$

The potential Φ is normalized to the electron temperature divided by elementary charge kT_e/e_0. The components of the ion velocity are normalized to the so called normalizing velocity c_0, defined in (13). The space coordinate x is normalized to the Debye length λ_D. The difference between the normalizing velocity c_0 and the ion sound velocity c_S should be emphasized. The ion sound velocity is defined as:

$$c_S = \sqrt{\frac{kT_e^* + \kappa kT_i}{m_i}}. \tag{14}$$

Taking into account that the screening temperature T_e^* is equal to the electron temperature T_e because there is only one electron population present in our model and that $\kappa = 1$ and using (13) the ions sound velocity is written in the following way:

$$V_S = \frac{c_S}{c_0} = \sqrt{1+\Theta}. \tag{15}$$

Using equations (8) - (12) the quasi neutral pre-sheath region and the sheath region can be analyzed together. But sometimes one is interested only in the neutral pre-sheath region. In this case the Poisson equation (12) is replaced by the neutrality condition:

$$n\left(X\right) = \exp\left(\Psi\left(X\right)\right). \tag{16}$$

The system of equations (8) - (11), (16) is then used for the analysis of the pre-sheath only.

3 Results

In this section some results of the model described in the previous section are presented. The presentation is started with the analysis of the pre-sheath region only. The system of equations (8) - (11), (16) is solved. The system of equations (8) - (11), (16) is highly nonlinear and must be solved numerically. The integration is started at $X = 0$ and progresses in the positive X direction towards the electrode. One has to deal with an initial value problem. So the initial conditions, i. e. the initial values of the functions Ψ, n, V_x, V_y and V_z have to be selected. The following initial conditions are selected:

$$\begin{gathered} n\left(0\right) = 1,\ \Psi\left(0\right) = 0,\ \frac{\mathrm{d}\Psi}{\mathrm{d}X}\left(0\right) = 0, \\ V_x\left(0\right) = V_0,\ V_y\left(0\right) = 0,\ V_z\left(0\right) = 0. \end{gathered} \tag{17}$$

It is assumed that at the initial point $X = 0$ the plasma is not perturbed, so the plasma density there is n_0, which gives the first initial condition. In the unperturbed plasma also electric field must be zero and this gives the third initial condition. The second initial condition simply tells that the plasma potential at $X = 0$ is selected as the zero of the potential. In our model the ions are born at rest so all three velocity components should be zero. But in this case only the zero solution of the system can be found. So a small initial velocity [1,3,4] in the positive X direction must be selected. A typical value is $V_0 = 10^{-8}$. In Fig. 2 the potential $\Psi(X)$ and velocity $V_x(X)$ profiles are shown. The parameters are the following: $K = 50$, $Z = 2$, $\alpha = 20^o$, $\varepsilon = 10^{-4}$ and 3 values of Θ are selected: $\Theta = 0$, $\Theta = 1$ and $\Theta = 2$.

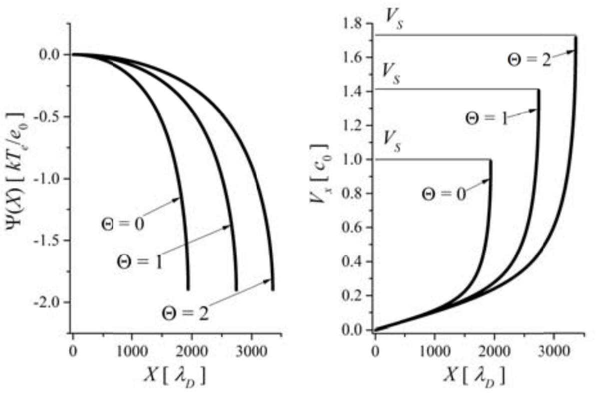

Figure 2: Potential $\Psi(X)$ and velocity $V_x(X)$ profiles obtained from the system (8) - (11), (16) for the parameters: $K = 50$, $Z = 2$, $\alpha = 20^o$, $\varepsilon = 10^{-4}$, $\Theta = 0$, $\Theta = 1$ and $\Theta = 2$ and initial conditions (17) with $V_0 = 10^{-8}$.

It can be seen that the pre-sheath length increases with increasing Θ while the potential at the sheath edge remains constant. In Fig. 3 the position of the sheath edge X_{SE} and the ion sound velocity V_S are plotted versus Θ.

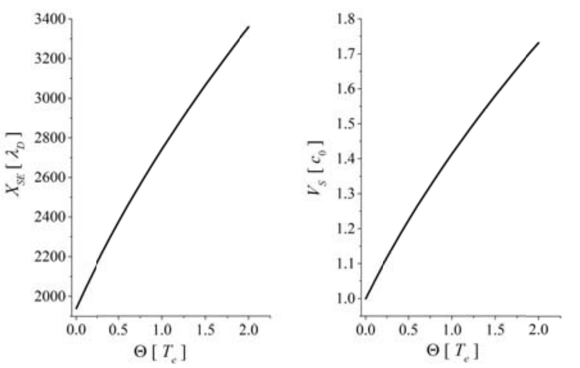

Figure 3: The sheath edge position X_{SE} and the ion sound velocity V_S versus Θ.

We now move to the analysis of the sheath and pre-sheath together, using the system (8) - (12). In Fig. 4 potential $\Psi(X)$ and velocity $V_x(X)$ profiles obtained from the system of equations (8) - (12) are shown. The following parameters are selected: $K = 50$, $Z = 2$, $\alpha = 20^o$, $\varepsilon = 10^{-4}$ and $\Theta = 0$. The initial conditions (17) are used, with $V_0 = 10^{-8}$.

Comparison of Figs. 2 and 4 shows an important difference between the solutions on the system (8) - (11), (16) on one hand and of the system (8) - (12) on the other. The solution of the pre-sheath system (8) - (11), (16) breaks down when the ion velocity V_x reaches the ion sound velocity V_S. The sheath edge position and consequently also the potential can therefore be determined from the following condition:

$$V_x\left(X_{SE}\right) = V_S = \sqrt{1+\Theta}. \qquad (18)$$

Equation (18) is the well known Bohm criterion [8].

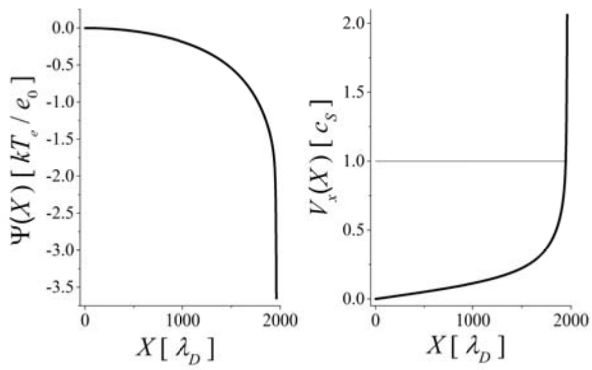

Figure 4: Potential $\Psi(X)$ and velocity $V_x(X)$ profiles obtained from the system (8) - (12) for the parameters: $K = 50$, $Z = 2$, $\alpha = 20^o$, $\varepsilon = 10^{-4}$, $\Theta = 0$ and initial conditions (17) with $V_0 = 10^{-8}$.

The solutions of the system (8) - (12) on the other hand can be continued to arbitrary X. A natural question arises where should the integration of the system (8) - (12) be stopped. One possibility is the floating potential of the electrode. At the floating potential the ion and electron current to the electrode are equal,

$$j_i = j_e. \qquad (19)$$

Here j_i is the ion current density and j_e is the electron current density. The ion current density is given by the product of the ion velocity and density:

$$j_i = u_{ix} n_i. \qquad (20)$$

The electron current density is calculated from the Maxwellian velocity distribution function:

$$j_e = n_0 \exp\left(\frac{e_0 \Phi(x)}{kT_e}\right)\sqrt{\frac{kT_e}{2\pi m_e}}. \qquad (21)$$

Combining (13), (19), (20) and (21) the floating condition (19) is written in the following form:

$$n(X) V_x(X) = \frac{1}{\sqrt{2\pi\mu}} \exp\left(\Psi(X)\right). \qquad (22)$$

From (22) a so called floating function $\Psi_{flt}(X)$ is defined in the following way:

$$\Psi_{flt}(X) \equiv \ln\left(n(X) V_x(X) \sqrt{2\pi\mu}\right). \qquad (23)$$

The recipe how to find the floating potential is now obvious. One must find either the point X and the respective $\Psi(X)$ where (22) is fulfilled or the intersection between the potential profile $\Psi(X)$ and the floating function $\Psi_{flt}(X)$ defined in (23). Both methods for determination of the floating potential are illustrated in Fig. 5. The following parameters are selected: $Z = 2$, $\varepsilon = 10^{-5}$, $K = 50$, $\alpha = 20^o$, $\Theta = 0$, $V_0 = 10^{-8}$, $\mu = 1/100$ and the initial conditions (17) are used. In the top two plots the ion and the electron flux are plotted and in the bottom plots the potential profile and the floating function are displayed. In the left graph a part of the respective right plot is shown on an expanded scale.

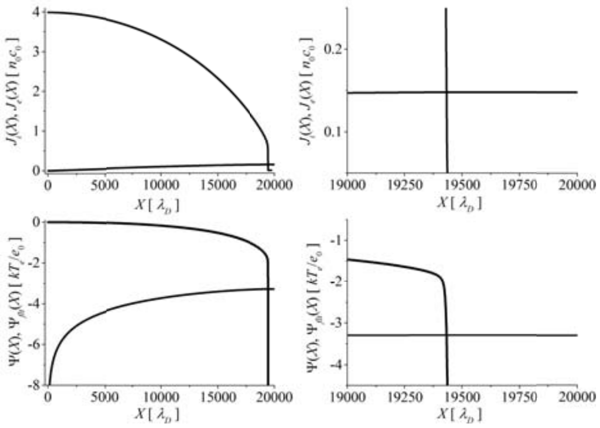

Figure 5: The ion and electron flux are shown in the top graphs. In the bottom plots the potential and the floating function are plotted. In the right plot a part of the respective left figure is shown on an expanded scale.

The floating potential and the respective coordinate can be determined using (22) or (23), while the sheath edge coordinate and potential can be determined from the Bohm condition (18).

So for a given set of parameters the sheath thickness and the potential drop in the sheath can be found easily. An example is shown in Fig. 6. The following parameters are selected: $Z = 2$, $\varepsilon = 10^{-5}$, $K = 200$, $\Theta = 0$, $\mu = 1/3670.48$ (deuterium mass)

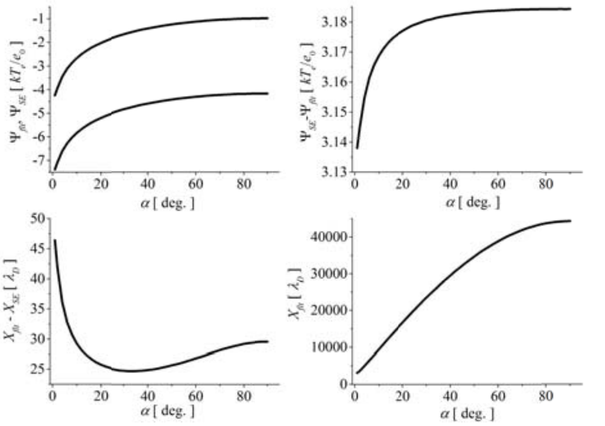

Figure 6: The floating and the sheath edge potential (top left), the sheath potential drop (top right), the sheath thickness (bottom left) and the electrode position (bottom right) versus the magnetic field angle α.

The initial conditions are given by (17) with $V_0 = 10^{-8}$. With α increasing the floating potential, the sheath edge potential and the sheath potential drop all increase, while the sheath thickness has a minimum.

4 Conclusions

A one-dimensional fluid has been presented and used for the analysis of the potential formation in front of a negative planar electrode immersed in a magnetized plasma. The magnetic field lines form an arbitrary angle with the electrode surface. The only exception is that they must not be completely parallel to the electrode surface. From the model equations the potential and velocity profiles can be found numerically. When they are calculated also the sheath edge coordinate and potential can be identified using the Bohm condition (18) and also the floating potential and the respective coordinate can be found using (22) or (23). From this the sheath thickness and the sheath potential drop can be calculated. With increasing magnetic field angle the sheath edge potential, the floating potential and the sheath potential drop all increase, while the sheath thickness has a minimum. The effects of the non-zero ion temperature have been studied. If the ion temperature is increased, only the pre-sheath length increases, while the pre-sheath potential drop remains constant. Comparison of the predictions of our fluid model with particle-in-cell computer simulations is under way.

Acknowledgements

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