Software Application for Solving the Transportation Problem

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Abstract

In this paper, we firstly describe a general model of the transportation problem, as well as some methods for determining the so-called initial basic solutions and optimization of transportation programs. The main part of the paper contains algorithms and software for implementation of the above solving methods within the original software package, in the form of MS Access database, and written in VBA (Visual Basic for Application) programming language. It represents a complete and comprehensive software tool which completely solves the transportation problem.

1 Introduction

Problems related to the transport problem are mathematically formalized a long time ago. First of all, L. Kantorovich in 1939 published the first results of research in the organization and planning of production. Then, a lot of research involved in this issue, especially during the Second World War. (It was used to determine how to move troops, located at training basis in different parts of the United States, to battlegrounds in Europe and Asia.) During this period, F. Hitchkok [4] gives the first mathematical formulation of transportation model. Later, G. Dantzig [3] formulates the transportation problem as a special problem of linear programming. Later on, many authors made a significant scientific contribution to the development of new ideas and methods to solve different aspects of the transportation problem. Today, the basic mathematical settings of this problem enables effectively solve a number of related issues, such as problems of allocation and scheduling, as well as the dynamic routing in telecommunication networks (see, e.g. Ash [1]).

2 Mathematical formulation of the transportation model

In its general form, the transportation problem is a mathematical model which determines the optimal program of transportation certain quantities of homogeneous goods (sources) $A_1, ..., A_m$ in the so-called demand centers (origins) $B_1,..., B_n$. The main criterion of optimization is the minimization of the total cost of transportation of goods, although it can be taken as a criterion the reduction of other parameters (total time of transport of goods, the degree of commitment of funds, etc.). In doing so, we assume that there is a physical separation between the sources in relation to centers of demand, and a total $m \cdot n$ different ways in which goods can be delivered. In that way, the transportation problem has a natural network representation, as is shown in the Fig. 1.
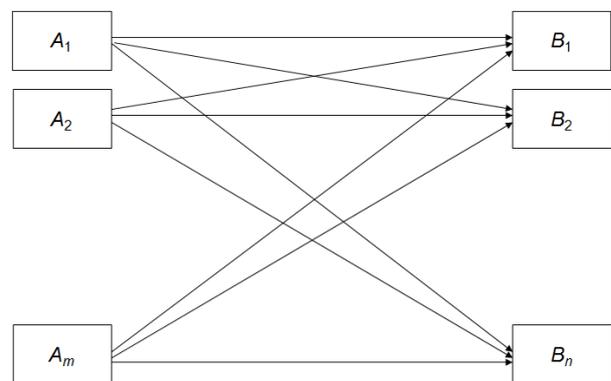

Figure 1. Diagram of all possible transportation routes of a homogeneous goods

In order to formulate a general, mathematical form of transport model, here are some of its key elements:
• Available quantities of manufactured goods (supply): $a_1,..., a_m$.
• The needs of the consumer centers (demand): $b_1,..., b_n$.
• Transportation costs per unit of output from the source A_i in the origin B_j. We shall denote those costs as c_{ij}, where $i = 1,..., m$, $j = 1,..., n$.
• Quantities of goods transported from the i-th source to the j-th origin. We shall denote them as x_{ij}.

The main goal of solving the transportation problem is finding the optimal values of x_{ij}, for which the total costs of transportation, defined by so called *the transportation function*

$$T = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad (1)$$

be the smallest. In addition, the following limiting conditions must be satisfied

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad \sum_{i=1}^{m} x_{ij} = b_j. \qquad (2)$$

This means that the total supply of each source must be fully allocated to the places of demand, while demand

for each demand centre must be completely fulfilled. Naturally, the amount of goods that define the variables x_{ij} are nonnegative and unknown, e.g. it holds $x_{ij} \geq 0$, for any $i = 1,..., m$, $j = 1,..., n$.

The transportation function (1) along with the above the constraints (2) defines a so called general (mathematical) form of the transportation problem (TP). It is a special form of linear programming (LP) problem. The achievement of minimum transportation costs described transport functions (1) can be interpreted as a minimal problem in the "classical" task of LP. However, the very specificity of this model allows us a number of different, simplified procedures for its solution. First of all, the TP can be displayed in a clear manner using so called the standard transportation matrix, shown in Figure 2.

Destinations → Sources ↓	B_1	B_2	...	B_n	Σ
A_1	c_{11} x_{11}	c_{12} x_{12}	...	c_{1n} x_{1n}	a_1
A_2	c_{21} x_{21}	c_{22} x_{22}	...	c_{2n} x_{2n}	a_2
...	...	...	...	...	...
A_m	c_{m1} x_{m1}	c_{m2} x_{m2}	...	c_{mn} x_{mn}	a_m
Σ	b_1	b_2	...	b_n	S

Figure 2. Standard transportation matrix

This matrix is often used for a more user friendly presentation and efficient solving of specific problems and tasks. On the basis of the above-defined models can be prove the following statement which characterizes the necessary and sufficient conditions of solvable TP.

Theorem 1. The transport problem has a solution if and only if the total supply equals the total demand, i.e.

$$\sum_{j=1}^{n} a_i = \sum_{i=1}^{m} b_j = S . \qquad (3)$$

The proof of the previous (and the following) theorem can be find, for instance, in [1]. The TPs in which it is satisfied the equality (3) are called *the closed transport problems*. Also, it is valid

Theorem 2. In the closed TP, the system of equations (3) has $m + n - 1$ linearly independent equations, i.e. coefficient matrix of this system has rank $m + n - 1$.

Based on the previous theorems it can be concluded that any solution and TP has the same number of values of x_{ij}, exactly $m + n - 1$, which will be different from zero. We'll call this value *a basic values*, unlike other, *non-basic values* for which it is valid $x_{ij} = 0$. The main goal in solving TP is to find the optimal values x_{ij} whose provide minimum total cost of transportation of goods, i.e. values for those the transportation functions (1) reaches its minimum.

3 Software implementation of the transportation problem

The process of determining the optimal values x_{ij} is usually done in two stages:
1. Determination of an initial solution of the TP.
2. Transformation of the initial solution to the optimal solution, i.e. transportation program with minimum total cost.

In this way, there may be different ways of determining the initial and the optimal solution. In the following section will be described in more detail some of them.

3.1 Determination of the initial basic solution

Formation of initial basic solution is an initial procedure of placement the specified quantity of goods that will satisfy the given conditions of TP, mainly the system of constraints (2). The main goal is to create one of the possible solutions to the TP, i.e. the total amount of goods distributed by the individual outcomes to appropriate destinations. At the same time, it must be satisfied to offer outcomes and destination demand.

The initial basic solution to the transportation problem can be determined in different ways. Here we give some of the most common methods, along with their software implementation, where we point out that the quality of the obtained solutions.

A. *North-west corner method*, also known as *diagonal method*, is one of the simplest methods of finding the initial solution. Assignment of goods is done from the top left corner of the matrix of transport, i.e. from the fields with coordinates (1, 1). Arrange of goods is carried out so that either completely exhausted offer outcomes or destination demand. Then, depending on whether it is fully satisfied demand or offer, to be completed next adjacent field. The basic algorithmic steps for this procedure consists the following programming elements:

Step 1: Input the number of sources (m) and the number of destinations (n).

Step 2: Input the values of a_i, b_j and c_{ij}, $i = 1,..., m$, $j = 1,..., n$.

Step 3: For $i = 1,2,...,m$ and $j=1,2,...,n$ repeat the following steps.

Step 4: If $a_i < b_j$ Then $x_{ij} = a_i$ Else $x_{ij} = b_j$.

Step 5: If $x_{ij} > 0$ Then
$T = T + c_{ij} * x_{ij}$; $a_i = a_i - x_{ij}$; $b_j = b_j - x_{ij}$.

Step 6: Show the values of T and x_{ij}, respectively.

B. *The minimal cost method,* as its name suggests, is based on the overarching filling the fields with the

lowest coefficient of transportation costs c_{ij}. In this way, in the most $m+n-1$ steps are obtained initially basic solution, i.e. the program of transportation in which each basic value $x_{kl} > 0$ value is obtained by the relation

$$c_{kl} := \min_{i,j}(c_{ij}) \quad \Rightarrow \quad x_{kl} := \min(a_k, b_l). \qquad (4)$$

The resulting solution usually has lower transport costs compared to the solution obtained by the previous one, the diagonal method. Figure 3 illustrates a part of VBA programming procedure by which this method is implemented in a software environment.

Figure 3. VBA procedure for the minimal costs method

C. _Vogel's approximate method,_ also known as the _method of the biggest differences_, certainly is the highest quality, but also the most complex method of determining the initial of the basic solution. Its basic idea is to calculate the potential losses incurred if the inside row, i.e. the column of the matrix of transport, instead of for at least one coefficient c_{ij} in basic solution include field with higher transportation costs. In other words, for each source and destination is calculated potential (opportunity) loss as the difference of the two lowest coefficients c_{ij}. Row or column with the biggest difference has the largest potential losses, and the first to be filled, starting from the element with the lowest coefficient of transportation costs. In each subsequent step, we repeat the same procedure and, thus, we get the initial basic solution enables of the TP. In the same way as the previous, the procedure described above can be easily shown in the form of algorithms, and the appropriate programming procedure to implement and solved using computer (Fig. 4).

3.2 Optimization of the transportation program

Determination of the initial basic solution, as we have already pointed out, is only a first initial step in solving the TP. In the next step, obtained solution is improved (optimized) so that the total transportation costs, expressed by the transportation function (1), are minimal. In this way, an optimal solution of the TP,

with the lowest total cost of transportation of goods is obtained. Procedures for obtaining the optimal solution may be different. However, all optimization methods must satisfy two basic criteria:
(_a_) Checking the optimality of the current basic solutions.
(_b_) Provide methods which will allow improving the current program of transportation, and creating a new program, with reduced total cost of transportation.

From the viewpoint of practical applications, optimization is usually performed in a series of iterative steps that enable successively, alternately perform both of these criteria.

Figure 4. VBA procedure for the Vogel's method

The two most common methods of optimization, which are usually used, are the following:
1. _Stepping-stone metod_ is based on the calculation of so called _coefficient of relative costs_ (Δ_{ij}), i.e. the formation of possible redistribution of goods from basic (completed) field (i,j) on a non-basic (empty) fields $(i_1,j_1),\ldots, (i_k,j_k)$ of the current program of transport. The coefficients Δ_{ij} are determined by the relative costs of non-basic fields and obtained successively adding and subtracting the unit costs, by the relation

$$\Delta_{ij} = c_{ij} - c_{i_1 j_1} + c_{i_2 j_2} - \cdots - c_{i_k j_k}. \qquad (5)$$

Thus values indicate how much will change total transport costs if the field (i,j) be included in the current program of transport. Software procedure designed for solving this problem is more complex compared to the previous one. Due to its complexity will be left out a detailed description of its structure. Though, in Figure 5 can be seen a small part of this procedure, where is shown the calculation of the coefficients Δ_{ij}.

2. _Method of potentials (MODI method)_ was developed by Dantzig as a particular method of solving linear programming problems. The basic idea is that each source and destination assignment so called dual variables (simplex multipliers) U_i, V_j. Based on these values may be, as in the previous method, calculated the

Figure 5. VBA procedure for Stepping-stone method

Figure 6. VBA procedure for the method of potentials
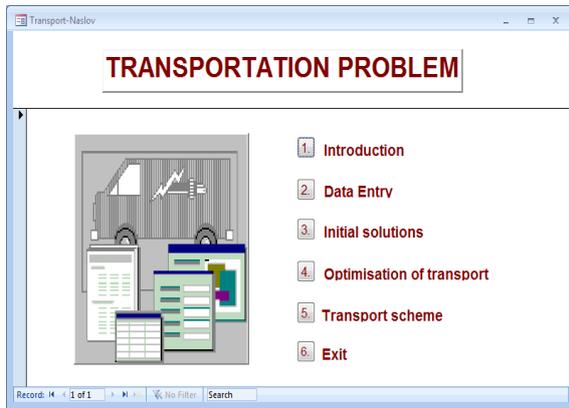
Figure 7. The switchboard form of the application

Figure 8. Output results and the optimal solution of the TP

coefficients of relative costs, and get improvement program of transportation. The part of the VBA procedure which calculates the simplex multipliers and determines the optimal solution based on this method is shown in Fig. 6.

Finally, Figure 7 shows the main, switchboard form of the software application which solves the TP. Here, the user can be informed about the model of TP, and the various ways of solving this problem. Then, he can select the option to enter data, determining the initial and optimal solutions. For this purpose, it is necessary to appoint the one of the three methods mentioned above, intended for finding an initial solution of the TP. After that, similarly, he may choose the procedure of optimization, selecting one of two possible methods that we described earlier. Finally, the software shows in a clear way, in the form of so-called transportation scheme, the appropriate outputs, i.e. an optimal program of transportation, as well as a minimum total cost of transportation (Fig. 8).

4   Conclusion

In this paper were described only few of the lots of possibilities of applications the computer-aided solvers in solving TP, as an important econometric problem. Most of these problems, by their characteristics, belong to the areas of (linear or nonlinear) programming (see,

for the most details, e.g., [5]). In that way, solving of these kinds of problems is closely directed to the appropriate software implementation (see, for instance, [6], [7]).

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