Combining Combinatorial Methods and Block-Based Crossover to Improve Unit Commitment Solutions

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Abstract. *The Unit Commitment (UC) problem is a key aspect of operational planning in power systems as it involves determining the on/off scheduling of thermal units to minimize total generation costs. In this paper, a combinatorial method based on variations with repetition is applied to generate feasible UC solutions, followed by hourly economic dispatch. The solution is iteratively created by accepting only those states that fulfil all system and technical constraints for each hour of the planning horizon. To improve the initial feasible solutions, we introduce an hourly block-based crossover procedure. For a selected hour, the states of all thermal units are exchanged between two solutions. A new solution is only accepted if it is feasible and has a lower total cost. Applying this method to a test system with ten thermal units and a planning horizon of 24-hours shows that the block-based crossover can effectively improve suboptimal solutions with minimal computational effort. This makes the method suitable for integration into more comprehensive heuristic or metaheuristic optimization methods.*

1 Introduction

Improving solutions to the UC problem remains a significant challenge in the operational planning of power systems. The UC problem involves determining the optimal switch on/off of thermal units over a discrete time horizon such that power demand is met at minimum total cost. This optimization must take into account a variaty of system and technical constraints, including generation limits, minimum on/off times, ramp rates, start-up costs, and reserve requirements. The complexity of the UC problem increases exponentially with the number of units and the length of the planning horizon, leading to a combinatorial explosion of possible unit states. Consequently, the search for optimal or near-optimal solutions requires the application of efficient optimization methods.

Various techniques for solving the UC problem have been studied in the literature. The most commonly discussed deterministic approaches include branch-and-bound [1], dynamic programming [4], priority list methods [6], Lagrangian relaxation [12], and mixed-integer programming [10]. In addition to deterministic techniques, various heuristic methods are widely used. Genetic algorithms (GA) [11] are among the best known, followed by particle swarm optimization (PSO) [3], ant colony optimization (ACO) [7], simulated annealing, artificial neural networks, expert systems, evolutionary programming. Moreover, hybrid approaches combining deterministic and heuristic strategies have also been proposed to utilize the strengths of both paradigms [5].

In this paper, a combinatorial approach for generating feasible UC solutions based on variations with repetition is proposed. The initial solutions generated in this way are guaranteed to fulfil all relevant constraints. To further improve these feasible solutions, we introduce an hourly block-based crossover technique as a local improvement method. This technique iteratively swaps the block state vectors for selected hours between two proposed solutions, accepting the new configuration only if it remains feasible and leads to lower overall generation costs. The main goal is to achieve cost-efficient, constraint-compliant UC solutions with minimal computational complexity, making the method suitable for integration into larger metaheuristic frameworks.

2 Problem Formulation

The UC problem addresses the scheduling of thermal power units over a discrete scheduling horizon, typically 24 hours, by determining both the commitment status and generation levels of each unit for each time period. The primary objective is to minimize the total generation cost while satisfying a set of operating constraints at both the system and unit level. Formally, the UC problem involves two types of decision variables:

1. Binary variables $u_i \in \{0,1\}$, which indicate the on/off status of thermal unit i at hour t;
2. Continuous variables $P_i(t)$, which indicate the active power of unit i at time t, provided it is switched on.

The objective function aims to minimize the total cost over the scheduling horizon, which includes fuel and start-up costs:

$$\min \sum_{t=1}^{T} \sum_{i=1}^{N} \left[F_i\big(P_i(t)\big) \cdot u_i(t) + SC_i(t) \right] \qquad (1)$$

where:

- $F_i\big(P_i(t)\big)$ is the fuel cost function of unit i at hour t,
- $SC_i(t)$ represents the start-up cost (either hot or cold) depending on the unit's prior status.

Since the UC problem is a mixed-integer, non-linear optimization problem, numerous constraints must be considered such as the power balance between demand and generation; the spinning reserve requirements; the minimum and maximum generation limits for each unit; the minimum start-up and shutdown time; the ramp rates; the start-up/shutdown curves and the associated costs. The complete mathematical formulation of the UC problem, including all relevant constraints and modelling assumptions, can be found in the standard works [2, 5]. Once the objective function and constraints are defined, the next step is to select a suitable solution method that can efficiently handle the combinatorial and continuous aspects of the problem.

3 Solution Methodology

The solution of the UC problem requires simultaneous decisions on the switch-on and switch-off state of the thermal units and the allocation of their active power output in order to minimize the total generation costs. These decisions must be consistent with a large number of system-and unit-specific constraints. A detailed combinatorial formulation and previous work dealing with this approach can be found in [8, 9].

This work focuses on improving already realisable UC solutions by applying an hourly block-based crossover strategy. Instead of performing a complete re-optimization, the proposed method works as a local search mechanism that aims to reduce the generation cost through targeted changes. Specifically, it involves the exchange of blocks corresponding to individual hours between two feasible proposed solutions. This localized exchange process enables efficient exploration of the solution space with minimal computational effort, while ensuring that all operational constraints remain satisfied. The entire algorithmic process is illustrated in Figure 1 using a flowchart. The methodology is particularly well suited for integration into more comprehensive heuristic or metaheuristic procedures, as it provides a balance between solution quality and execution time.

3.1 Initial Solution

The proposed methodology starts with two feasible UC solutions. Each solution satisfies all necessary constraints, including power balance, minimum up and down times, block power constraints, spinning reserve commitments, and start-up costs. Each solution can be represented as a binary matrix with dimensions $T \times N$, where T is the number of time intervals (usually hours) and N is the number of thermal units. The value of each element $u_i(t) \in \{0,1\}$, indicates the operating status (ON/OFF) of unit i at hour t.

3.2 Hourly Block-Based Crossover Method

To improve the quality of the initial feasible solutions, an hourly block-based crossover strategy is applied as a local search operator. The procedure is defined as follows:

- For each hour $t \in \{1,2,\dots,T\}$ the UC states from both solutions are compared;
- If the states for hour t are identical, the algorithm continues with the next hour;
- If differences are found, a block swap is attempted, i.e. the entire column for hour t is swapped between the two solutions;
- Two new candidate solutions are generated from this swap.
- Each new solution is then evaluated:
  - *Feasibility Check:* does the solution fulfil all UC constraints?
  - *Cost Evaluation:* does the solution result in lower total generation costs?

  If both conditions are met, the new solution is accepted and used as the basis for further crossover operations in subsequent hours.

This process is continued iteratively for all time intervals until no further improvement is observed. The method ensures that local refinements are applied without violating feasibility, making it a lightweight and effective improvement to combinatorially generated solutions.

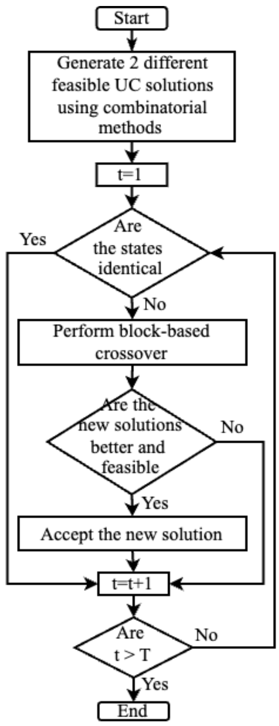

Figure 1. Flowchart of the proposed hourly block-based crossover method.

3.3 Visual Illustration of the Method

To facilitate the understanding of the proposed block-based crossover mechanism, a visual illustration of the main steps involved in the exchange process is given in this section. The illustrations are designed to be applicable for any number of thermal units and time intervals.

In Figure 2, two feasible UC solutions are shown side by side, with a specific hour (e.g., the third hour) highlighted as the target for block exchange. Each coloured block represents the commitment status of all units at that hour.

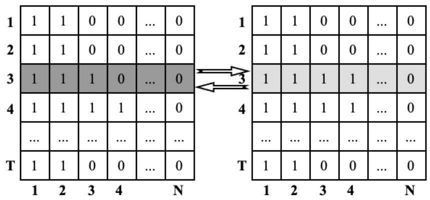

Figure 2. Block selection for crossover.

The exchanged blocks are reinserted into the opposing matrices, creating two new solutions. After this exchange, the two newly formed solutions are subjected to a standard feasibility and cost assessment. If a solution remains valid and leads to a reduction in overall generation costs, it is retained for further improvements in subsequent iterations.

This visualization provides an intuitive overview of the crossover mechanism and its local optimization effect. It can be easily integrated into heuristic or metaheuristic methods without the need for additional mathematical formalism or high computational complexity.

4 Results

The proposed block-based crossover method was evaluated on a standard test system consisting of ten thermal generation units (see Table 3) over a 24-hour planning horizon with predefined hourly load values according to [2].

The initial solutions (IS) were generated using a combinatorial method that ensures feasibility with respect to all operational constraints. Using variations with repetition, where $n = 2$ (operational state {0,1} of a thermal unit) and $N = 10$ (the number of thermal units), the total number of combinations we have to try for each hour is $2^{10} = 1024$, but most of them are not suitable for use in a power system due to the impossibility of loading. In a previous work [9], the exact number of variations with repetition used in a power system for different loads occurring in the system was shown.

The block-based crossover algorithm was then applied to these two feasible solutions by iteratively comparing and attempting to swap the UC blocks for each hour. Table 1 shows the total generation costs for the initial and final solutions.

Table 1. Generation costs before and after crossover

Solution	Total Cost ($)
IS 1 (Feasible)	564485
IS 2 (Feasible)	563977
After Block-Based Crossover	**563937**

As shown, applying the crossover method resulted in a new solution with a lower total cost than the two initial feasible solutions. During the search process, a total of 24 crossover attempts were made, one for each hour. However, only two of these crossovers (in hours 4 and 23) resulted in feasible and cost-improving solutions. Since the solutions only differed in the 4th and 23rd hours, the better solutions were taken from these hours, while maintaining the feasibility of the overall solution.

This result shows that although the block-based crossover does not guarantee an improvement in every iteration, it can serve as an effective local search operator with minimal computational cost. A detailed example of the operating and start-up costs, hourly load and generation schedule using the described approach is shown in Table 2.

5 Conclusion

An efficient block-based crossover method has been proposed to improve feasible UC solutions. By exchanging hourly unit schedules locally, the approach reduces the generation cost while maintaining feasibility. The results confirm its effectiveness with low computational cost. Future work will focus on multi-hour exchanges and integration with metaheuristic systems to improve optimization performance.

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Table 2. Hourly operation summary: costs, load, and generation schedule

Hour	**Operation cost ($)**	**Start-up cost ($)**	**Load (MW)**	**Generation schedule**									
				1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	13683	0	700	455	245	0	0	0	0	0	0	0	0
2	14554	0	750	455	295	0	0	0	0	0	0	0	0
3	16809	900	850	455	370	0	0	25	0	0	0	0	0
4 (IS 2) ✓	18598	0	950	455	455	0	0	40	0	0	0	0	0
4 (IS 1) x	19146	560	950	455	340	0	130	25	0	0	0	0	0
5	20020	560	1000	455	390	0	130	25	0	0	0	0	0
6	22387	1100	1100	455	365	130	130	25	0	0	0	0	0
7	23262	0	1150	455	410	130	130	25	0	0	0	0	0
8	24150	0	1200	455	455	130	130	30	0	0	0	0	0
9	27251	860	1300	455	455	130	130	85	20	25	0	0	0
10	30058	60	1400	455	455	130	130	162	33	25	10	0	0
11	31916	60	1450	455	455	130	130	162	73	25	10	10	0
12	33890	60	1500	455	455	130	130	162	80	25	43	10	10
13	30058	0	1400	455	455	130	130	162	33	25	10	0	0
14	27251	0	1300	455	455	130	130	85	20	25	0	0	0
15	24150	0	1200	455	455	130	130	30	0	0	0	0	0
16	21514	0	1050	455	310	130	130	25	0	0	0	0	0
17	20642	0	1000	455	260	130	130	25	0	0	0	0	0
18	22387	0	1100	455	360	130	130	25	0	0	0	0	0
19	24150	0	1200	455	455	130	130	30	0	0	0	0	0
20	30058	490	1400	455	455	130	130	162	33	25	10	0	0
21	27251	0	1300	455	455	130	130	85	20	25	0	0	0
22	22736	0	1100	455	455	0	0	145	20	25	0	0	0
23 (IS 1) ✓	17645	0	900	455	425	0	0	0	20	0	0	0	0
23 (IS 2) x	17685	0	900	455	420	0	0	25	0	0	0	0	0
24	15427	0	800	455	345	0	0	0	0	0	0	0	0
Total: 559847+4090=563937 $													

Table 3. Thermal units parameters

No.	a_i	b_i	c_i	P_i^{min}	P_i^{max}	T_i^{on}	T_i^{off}	hc_i	cc_i	cs_i	**Initial state**	
1	1000	16.19	0.00048	150	455	8	8	4500	9000	5	8	+ (ON)
2	970	17.26	0.00031	150	455	8	8	5000	1000	5	8	+ (ON)
3	700	16.60	0.00200	20	130	5	5	550	1100	4	-5	- (OFF)
4	680	16.50	0.00211	20	130	5	5	560	1120	4	-5	- (OFF)
5	450	19.70	0.00398	25	162	6	6	900	1800	4	-6	- (OFF)
6	370	22.26	0.00712	20	80	3	3	170	340	2	-3	- (OFF)
7	480	27.74	0.00079	25	85	3	3	260	520	2	-3	- (OFF)
8	660	25.92	0.00413	10	55	1	1	30	60	0	-1	- (OFF)
9	665	27.27	0.00222	10	55	1	1	30	60	0	-1	- (OFF)
10	670	27.79	0.00173	10	55	1	1	30	60	0	-1	- (OFF)